Referee report (part I) on Voevodsky's C-system of a module over a monad on set
overall comments:
The first paragraph of the introduction lays out the laudable goal of the paper: to allow results about the semantics of particular dependent type theories to be derived as special cases from results of maximum generality. This paper is part of a series of papers by Voevodsky whose goal is to do that, and also to check the the semantics of a particular dependent type theory: Martin-Lof type theory with the Univalence Axiom. That type theory has been responsible for a vibrant renaissance of type theory, in which a new understanding of equality, as univalent equality, clarifies and strengthens the applicability of the type theory toward formalization of the proofs of modern mathematics. Checking the semantics means to provide a mathematical model for the type theory that proves that no contradiction is derivable within the theory, i.e., that the formalization techniques are sound.
In univalent type theory, mathematical objects are "elements" of "types". "Propositions" are defined to be those types whose elements are equal to each other, and "sets" are defined to be those types for which the type of equalities between any two elements is a proposition. That, along with the usual notions of "function" and "pair", allows the formalization of mathematical proofs. The univalence aspect is especially important, as it is the ultimate extensionality principle, from which all other known ones can be derived, and it opens up a new world where types that are not sets can be proven to exist. For example, if G is a group, then the type BG of all G-torsors will be a "connected" type whose "loop space" is the group G.
The current paper fulfills an important foundational role: to help put the notion of "model" of a type theory in an abstract framework. A model will be a functor from the "contextual category" associated to the type theory to another category, such as the category of simplicial sets. One must describe in precise terms how one passes from a list of derivation rules for "judgments" about formal expressions of the type theory to the contextual category. In this paper an intermediate structure is considered: a monad R : Sets -¿ Sets together with an R-module M : Sets -¿ Sets. For a set X, R(X) will be the set of formal "term" expressions with free variables drawn from the set X, and M(X) will be the set of formal "type" expressions. The monad structure on R comes from substitution of terms for the free variable in a term, and the R-module structure on M arises from substitution of terms for the free variables in a type.
Voevodsky presents the notion of "C-system", which is equivalent to the notion of contextual category. An initial "C-system" CC(R,M) is constructed from R and M, and then the syntactic judgments of the type theory being studied can be encoded as specifying a quotient of a subobject of CC(R,M).
Here are the papers in the series by Voevodsky currently visible on the arxiv:
1203.2553 - Univalence in Simplicial Sets, with Kapulkin and Lumsdaine 1211.2851 - The Simplicial Model of Univalent Foundations, with Kapulkin and Lumsdaine 1406.7413 - Subsystems and regular quotients of C-systems 1407.3394 - C-system of a module over a monad on sets 1409.7925 - A C-system defined by a universe in a category 1410.5389 - B-systems 1503.7072 - Products of families of types in the C-systems defined by a universe category 1505.6446 - Martin-Lof identity types in the C-systems defined by a universe category
For the purpose of understanding the importance of the current paper, which is 1407.3394, one presumably must understand how it fits into the whole program.
Probably the most interesting goal of the series of papers is to rigorously establish the soundness of type theory with univalence. The result is claimed in 1211.2851, but as Voevodsky later announced, certain details concerning the syntax of the derivation rules were not provided there. The purpose of at least some of the other papers in the series is to provide them; I suppose that 1410.5389 is not needed. I don't know whether 1503.7072 and 1505.6446 are needed, but I suspect that if they are

needed then many other similar papers covering various bits of the type theory (such as inductive definition) will be needed.

It's unclear to me why we need to construct C-systems both from monads and from universes, but I can guess: we can construct a C-system from our type theory by passing through monads, we can construct a C-system from simplicial sets by passing through universes, and then a model useful for verifying soundness will be a functor from the first C-system to the second C-system (satisfying certain other properties?). If something like that is right, it would be good if it were to be revealed in the introduction.

overall question 1:

Is the current paper, ”C-system of a module over a monad on sets”, envisioned to be an essential part of the eventual proof of soundness of type theory with univalence? I think it likely that the answer is yes. If so, I suggest stating that in the introduction and sketching the plan in two or three sentences, for that would add important motivation for the reader.

The old (incomplete) paper ”Notes on type systems”, now referred to as ”Old notes on type systems”, contains a sketch of the whole program, with the ambitious and worthy goal being to check the soundness of the type theory actually used in the software ”Coq” combined with the univalence axiom. There the older term ”triple” is used instead of ”monad”, and the material about it seems to be an essential part of the whole. So, unless something essentially new is planned, the answer to the question is still ”yes”.

overall question 2:

Which papers in the list above will be involved, in an essential way, in the proof of soundness of type theory with univalence?

I suggest adding the information to the introduction.

page 1:

In ”A modified axiomatics of C-systems and the construction of new C-systems as sub-objects and regular quotients of the existing ones in a way convenient for use in type-theoretic applications are considered in [15]” the subject of the sentence seems to be singular (a modified axiomatics) but the verb is plural (are).

page 2:

”modulo the -equivalence” - remove ”the”

page 3:

In the notation R_A,1 the 1 appears to play no role. I suggest explaining it or eliminating it.

page 3:

In the proof of 2.2 occurs the expression bind(f,pr_D(_(X,A))). I'm trying to check that it makes sense. Here f : X −¿ R_A,1(X'), and R_A,1(X') = pr_C(R(X',A)). And _(X,A) : (X,A) −¿ R(X,A), so pr_D(_(X,A)) : A −¿ pr_D(R(X,A)). Thus (f,pr_D(_(X,A))) : (X,A) −¿ (pr_C(R(X',A)),pr_D(R(X,A))). For bind(f,pr_D(_(X,A))) to make sense, we need (f,pr_D(_(X,A))) to be an arrow of the form (X,A) −¿ R(X”,A”), for some (X”,A”). But R(X,A) and R(X',A) both seems to be distinct from (pr_C(R(X',A)),pr_D(R(X,A))), unless X=X'. So now I don't know whether Lemma 2.2 is correct, as stated. I hope I'm not making some silly mistake.

Probably the general lemma closest to the statement of 2.2 would be the one that says this: Let C and D be categories. Let F:D−¿C and G:C−¿D be functors. Suppose F is left adjoint to D. Let R : C −¿ C be a monad. Then the composite GRF : D −¿ D has a natural structure of monad. I've checked enough details to render the statement at least plausible. The special case where R=1 is known and yields the monad on D associated to the pair G,F.

If that statement is true, it suggests that 2.2 is true under the additional hypothesis that A is an initial object of D. But later on, when 2.2 is applied, it seems that this additional hypothesis is not satisfied.

pages 3-4:
A module such as the one in the expression LM R LM seems to be called a right module, not a left module; see, for example, Definition 3.1 of reference [6], or http://ncatlab.org/nlab/show/module+over+a+monad . That's logical, anyway, because in the expression LM R, R appears on the right of the module. Variable names that look atomic would be better, so consider replacing "LM" by "M". And if you agree that LM is a right module, changing it to "RM" would introduce a partial conflict with "R".
page 4:
The notation "R cor" would look better with no spaces: "Rcor".
page 19:
I don't understand the remark "However, it does not allow to describe the substitution of, e.g., terms with one free variable into terms with one free variable". Isn't substitution adequately described by composition? ——————————-